

Effects of heat release on turbulent shear flows. Part 2. Turbulent mixing layers and the equivalence principle

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The general equivalence principle of Tacina & Dahm (2000) (Part 1) that extends scaling laws for non-reacting flows to account for density changes due to reaction heat release is applied to turbulent mixing layers to develop physically based scaling laws for heat release effects in exothermic reacting mixing layers. This leads to an ‘extended density ratio’ s^+ based on the equivalent elevated temperature for one of the two free-stream fluids that accounts for the density variations within the layer due to exothermic reaction. When used in place of the isothermal density ratio s in scaling laws for growth rate and entrainment ratio in non-reacting mixing layers, resulting predicted effects of heat release show good agreement with measured values, and reveal subtle effects of stoichiometry previously unnoticed in experiments. Results also suggest ways to achieve increased growth rates and entrainment ratios due to heat release in turbulent mixing layers. These results for heat release effects in mixing layers, and earlier results for heat release effects in the near and far fields of planar and axisymmetric jets, support the validity and utility of the equivalence principle between exothermic reacting turbulent shear flows and a corresponding equivalent non-reacting flow under otherwise identical conditions.

1. Introduction

Scaling laws for the effects of heat release on shear flows are essential for connecting the fluid dynamics literature on non-reacting flows to studies of reacting flows and applications in combustion. Tacina & Dahm (2000, herein referred to as Part 1) developed a general equivalence principle that allows heat release effects on the scaling of shear flows to be deduced from the scaling laws that govern the corresponding non-reacting flow. When applied to turbulent jets, this equivalence principle was shown to accurately predict heat release effects in the near and far fields of planar and axisymmetric turbulent jet flames. Here this equivalence principle is applied to turbulent mixing layers to develop scaling laws for predicting effects of heat release in turbulent mixing layers. Since experimental results documenting measured heat release effects on the entrainment and mixing properties of turbulent mixing layers are available for comparison, the present results also provide a further test of the validity and utility of the general equivalence principle in Part 1.

The mixing layer is one of the most widely studied of the classical turbulent free shear flows, due in part to its ubiquity in practical flow and mixing applications, especially those involving non-premixed or partially premixed combustion between two free-stream fluids. This interest has been further due to the prominent organized

large-scale vortical structure of the flow. Numerous studies have examined aspects of this organized structure in the mixing layer, and from these the mixing layer has assumed a wider role as a canonical turbulent free shear flow.

Those aspects of mixing layers having received significant attention include effects of the free-stream densities on the growth rate of incompressible non-reacting turbulent mixing layers. Brown (1974) used similarity arguments to derive an expression for the effect of the free-stream density ratio $s \equiv \rho_2/\rho_1$ and free-stream velocity ratio $r \equiv U_2/U_1$ on the growth rate $d\delta/dt$ of temporally developing mixing layers. Dimotakis (1986) subsequently obtained an analogous expression for the growth rate $d\delta/dx$ in spatially developing turbulent mixing layers. The latter shows good agreement with experimentally measured growth rates in isothermal mixing layers over a wide range of density ratios and speed ratios (e.g. Brown & Roshko 1974; Dimotakis 1991) and with numerical simulations of non-reacting turbulent mixing layers (e.g. Soteriou & Ghoneim 1995).

The growth rate of the turbulent mixing layer in turn determines the entrainment and mixing properties of the flow. Of particular importance for chemical reactions are the rates of entrainment of reactants carried by the two free-stream fluids into the layer, which are related to the growth rate and thus are affected by the free-stream density ratio s . Equally important for chemical reactions, the entrainment ratio E_v , giving the entrained volume coming from the high- and low-speed streams, typically differs significantly from unity and also depends on the free-stream density ratio s . The presence of such density effects even in isothermal mixing layers strongly suggests that density changes due to heat release will also affect the flow. The density changes that occur in exothermic reacting turbulent mixing layers produce local baroclinic generation of vorticity, as well as dilatation and other mechanisms, which collectively lead to the overall heat release effects for which scaling laws are developed here.

In exothermic reacting turbulent mixing layers, it is not immediately obvious if the effect of heat release should be to increase or decrease the growth rate of the flow. The dilatation due to exothermicity might suggest an increase in the growth rate with increasing heat release. However experiments by Wallace (1981) at relatively low levels of heat release, producing adiabatic temperature rises of less than 400 K, suggest precisely the opposite, namely a slight decrease in the layer growth rate with increasing heat release. Hermanson, Mungal & Dimotakis (1987) and Hermanson & Dimotakis (1989) subsequently used a hydrogen–fluorine mixing layer to produce adiabatic flame temperatures rises up to 940 K. Even with this larger heat release, they found that the growth rate of the layer decreased slightly with increasing heat release, up to 15 % at their highest flame temperatures. Their results also showed a reduction in both the volumetric entrainment rate and the entrainment ratio E_v into the layer with increasing heat release. In their study, the effects of heat release were interpreted in terms of the reduction in turbulent shear stresses within the mixing layer.

Several numerical studies have examined the effects of various elementary physical mechanisms that occur due to heat release, such a baroclinic generation and dilatation, on the detailed fluid dynamics within exothermic reacting turbulent mixing layers. Ghoniem, Heidarinejad & Krishnan (1988) have used a transport element method to investigate reacting mixing layers, and McMurtry, Riley & Metcalfe (1989) used direct numerical simulations (DNS) of a temporally developing turbulent mixing layer to investigate effects of heat release. Their results also confirmed the decrease in layer growth rate and entrainment ratio with heat release. They interpreted these effects in terms of turbulence energetics, vorticity dynamics, and stability theory. Sekar & Mukunda (1990) conducted a DNS study of compressible reacting turbulent

mixing layers with heat release, and also observed similar reductions in growth rate and entrainment rate due to heat release. Further numerical simulations by Son, McMurtry & Queiroz (1991) of a temporally developing mixing layer provided additional details regarding effects of heat release on statistics and spectra associated with various flow properties.

Additionally, Ganji & Sawyer (1980), Pitz & Daily (1983) and Keller & Daily (1985) have conducted experiments involving heat release by premixed combustion in turbulent mixing layers, and Ghoneim & Krishnan (1988) have conducted numerical simulations to study the mechanisms leading to heat release effects in such premixed reacting mixing layers. However the additional effects of strong pressure gradients in those experiments, as well as the important differences between premixed and non-premixed or partially premixed combustion, put such premixed cases outside the scope of the flows to be considered here. Similarly, while compressibility effects in mixing layers (e.g. Messersmith & Dutton 1986; Papamoschou & Roshko 1988; Papamoschou 1991; Goebel & Dutton 1991) also lead to density variations, the present results are limited to density effects produced by reaction heat release.

The findings of reduced growth rate and entrainment ratio in mixing layers due to heat release have significant implications for the development of combustors in high-speed propulsion systems. Much of the interest in heat release effects on turbulent shear flows stems from interests in developing propulsion systems for high-speed flight and next-generation reusable launch vehicles. The reduced growth rates and entrainment properties implied above in shear flows with heat release from highly exothermic reactions imply correspondingly longer combustors, with consequent penalties on other aspects of the overall system performance. Scaling laws for heat release effects in fundamental turbulent shear flows, including jets and mixing layers, may provide insights into ways to enhance the entrainment and mixing properties of practical shear flows undergoing large heat release.

This paper develops scaling laws for the overall effect of heat release due to nonpremixed or partially premixed reactions in incompressible turbulent mixing layers in the absence of any streamwise pressure gradient. It shows that the reduced growth rate and entrainment properties noted above, which result from density variations within the mixing layer produced by the temperature rise due to exothermic reaction, are quantitatively consistent with changes due to the free-stream density ratio s in non-reacting mixing layers. As suggested by the equivalence principle in Part 1, these density changes due to exothermic reaction can be related to an equivalent non-reacting mixing layer, in which the temperature of one of the free-stream fluids is raised to an effective value determined by the peak temperature and the overall stoichiometry. This leads to a generalized density ratio s^+ for mixing layers that extends previous results for the effects of s on properties of non-reacting mixing layers to allow prediction of heat release effects in exothermic reacting mixing layers. The resulting predicted effects of heat release show good agreement with experimentally measured values in mixing layers, and reveal subtle effects of stoichiometry not previously noticed in the experimental results. Additionally, the present results indicate how it may be possible to achieve increased growth rates and entrainment ratios due to heat release effects in turbulent mixing layers.

2. Scaling of non-reacting mixing layers

Turbulent shear flows in general vary much more slowly along the downstream direction than along the lateral direction, and for this reason are classically treated as

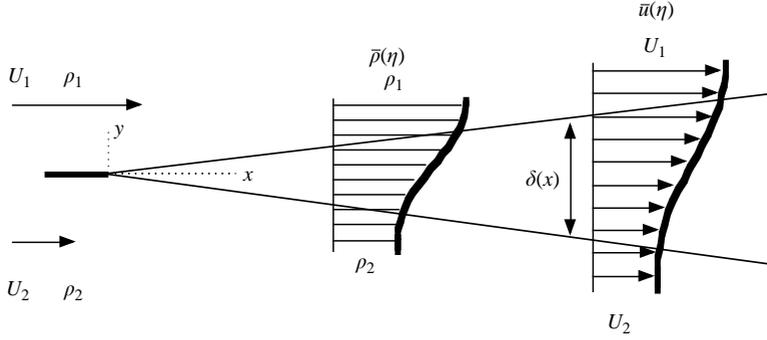


FIGURE 1. Notation for non-reacting turbulent mixing layers, showing mean velocity profile and mean density in terms of the similarity coordinate $\eta \equiv y/\delta(x)$.

quasi-one-dimensional. Local properties of the flow, such as entrainment and mixing rates, are thus determined by the local outer variables $\delta(x)$ and $u(x)$, namely the local length and velocity scales that characterize the local mean shear which drives the turbulence at any downstream location x in the flow. Scaling laws for δ and u can often be determined by simple dimensional reasoning, and in general depend on the fluid densities even in flows without heat release.

As noted in figure 1, in the self-similar far field of non-reacting turbulent mixing layers the outer velocity scale $u(x)$ is $(U_1 - U_2)$ and thus remains constant in the absence of any streamwise pressure gradient. The outer length scale $\delta(x)$ may be the visual thickness, the vorticity thickness, or any other definition that differs from these only in the proportionality constant. Dimensional considerations for non-reacting incompressible turbulent mixing layers indicate that the outer length scale $\delta(x)$, namely the local width of the turbulent mixing layer, must vary with downstream distance x , free-stream speeds U_1 and U_2 , and free-stream densities ρ_1 and ρ_2 as

$$\delta = xf(r, s), \quad (1)$$

where the free-stream speed ratio and density ratio are given by

$$r \equiv \frac{U_2}{U_1} \quad \text{and} \quad s \equiv \frac{\rho_2}{\rho_1}, \quad (2a,b)$$

with the subscripts 1 and 2 referring, respectively, to the high-speed and low-speed streams.

The growth rate $d\delta/dx$ of non-reacting mixing layers from (1) depends on the density ratio s . Density effects thereby play a direct role in setting the growth rate of the layer, even in the absence of heat release by exothermic reactions. For cases with a virtual origin x_0 , the apparent downstream distance is $(x - x_0)$ and the growth rate $d\delta/dx$ is equivalently expressed as $\delta/(x - x_0)$.

In spatially developing mixing layers, the growth rate can be obtained from the geometry of the large-scale organized structure of the flow (Dimotakis 1986) as

$$\frac{\delta}{(x - x_0)} \sim \left(\frac{1 - r}{1 + s^{1/2}r} - 1 \right) \left[(1 + s^{1/2}) - \left(\frac{1 - s^{1/2}}{1 + 2.9(1 + r)/(1 - r)} \right) \right], \quad (3)$$

where neglect of the term in the square brackets gives the corresponding growth rate for temporally developing mixing layers (Brown 1974). Note that the various definitions of δ differ only in (3) by the proportionality constant, which has no bearing on the present results.

As regards the volumetric entrainment ratio E_v that gives the volume ratio of high-speed to low-speed free-stream fluids being entrained into the layer, on symmetry grounds in temporally developing layers this must always be unity, and thus E_v in that case is independent of the density ratio. However in spatially developing layers, geometric considerations associated with the large-scale structure indicate that the flow typically entrains unequal amounts of the two free-stream fluids. The resulting entrainment ratio is given by (Dimotakis 1986)

$$E_v \approx s^{1/2} \left(1 + 0.68 \frac{1-r}{1+r} \right) \quad (4)$$

and thus like the growth rate also depends on the density ratio s , even in the absence of heat release by exothermic reactions.

It is apparent from the scaling laws in (3) and (4) that density effects in mixing layers influence the resulting growth rate and entrainment ratio, even in non-reacting layers, irrespective of whether the actual physical mechanism is by baroclinic generation, dilatation, or any other means. When heat release is present, the further density variations due to exothermicity are thus also expected to affect the flow as suggested by these scaling laws, again irrespective of the precise physical mechanism by which these changes are produced. The general equivalence principle in Part 1 provides the value for one of the two free-stream densities in a non-reacting flow that produces the same the density variations $\rho(\mathbf{x}, t)$ on either the rich or lean branch of the equilibrium temperature field $T(X)$ due to the mole fraction field $X(\mathbf{x}, t)$ in the exothermic reacting flow. The scaling laws for exothermic reacting mixing layers can thus be obtained from those in (3) and (4) for the corresponding non-reacting layers by replacing the actual density ratio s with the corresponding ratio s^+ for the free-stream densities implied by this equivalence. It will be seen herein that for mixing-limited exothermic reactions in turbulent mixing layers this simple equivalence provides scaling laws for such exothermic reacting mixing layers that correctly predict the effects of heat release on entrainment and mixing rates, as well as other flow properties that are determined by the outer variables.

3. Equivalence between exothermic and non-reacting flows

3.1. The equivalence principle

The equivalence principle introduced in Part 1 accounts for the effects of exothermic heat release in turbulent shear flows through the density in the scaling laws for the local outer variables that govern the corresponding non-reacting flow. The equivalence is based on the piecewise linear form of the equilibrium temperature $T(X)$ in the mole fraction field $X(\mathbf{x}, t)$ demanded by enthalpy conservation under the broadly applicable conditions noted in Part 1. Since a linear $T(X)$ is indicative of simple fluid mixing without reaction, it is apparent in figure 2 that on either side of the stoichiometric mole fraction X_s , the temperature field $T(X)$ in the reacting flow is equivalent to that which would occur in a corresponding non-reacting flow with the temperature of one of the fluids raised to a fictitious elevated value. This is equivalent to replacing one of the two fluid densities in the outer-variable scaling laws in the corresponding non-reacting flow with the effective value that corresponds to this elevated temperature. The density field $\rho(\mathbf{x}, t)$ in the equivalent non-reacting flow is then identical to that which occurs in the exothermic reacting flow wherever the mole fraction field $X(\mathbf{x}, t)$ is above (or below) the stoichiometric value X_s . In this manner,

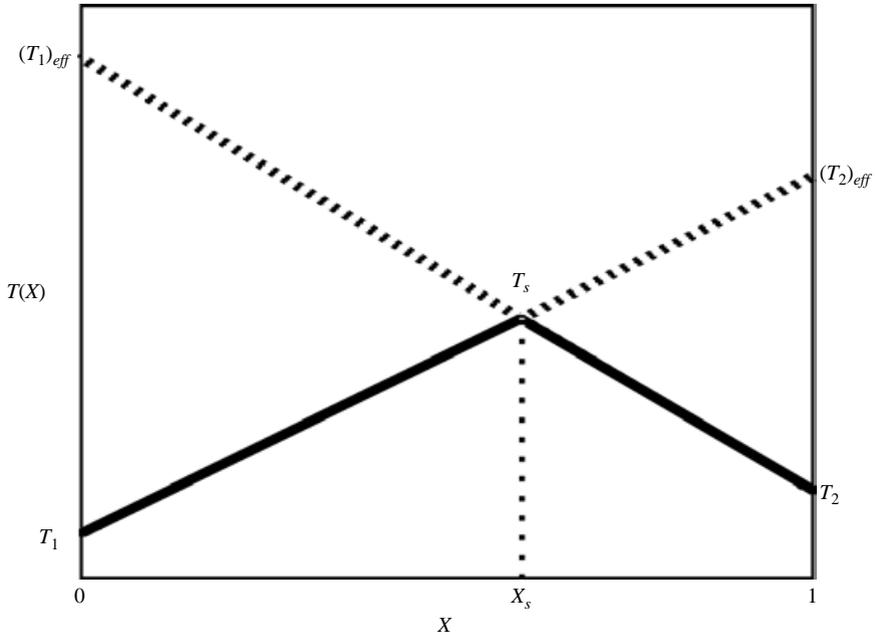


FIGURE 2. Temperature $T(X)$ in terms of low-speed mole fraction X , showing high- and low-speed free-stream temperatures T_1 and T_2 , stoichiometric mole fraction X_s and temperature T_s , and effective free-stream temperatures $(T_1)_{eff}$ and $(T_2)_{eff}$ in (5a) and (6a).

the dominant effects of density changes due to heat release in the exothermic flow are obtained from the scaling of the equivalent non-reacting flow.

Note that this does not require specifying whether baroclinic generation, dilatation, some combination of these or even some other means is the dominant elementary physical mechanism(s) in the flow that ultimately leads to the changes in growth rate and entrainment ratio due to heat release. Instead, the equivalence produces the density field $\rho(\mathbf{x}, t)$ in a non-reacting flow that most closely matches that in the exothermic reacting flow. Whatever the dominant physical mechanisms are that result from the density variations $\rho(\mathbf{x}, t)$ in the reacting layer, they will then also be present in the equivalent non-reacting layer wherever $X(\mathbf{x}, t)$ is above (or below) the stoichiometric value X_s .

Part 1 showed that this equivalence accurately predicts the dominant heat release effects in both the near and far fields of both planar and axisymmetric turbulent jet flames over a wide range of fuels and dilutions. In the far field of axisymmetric jets this led to a generalized momentum diameter d^+ in (11a, b) of Part 1 that extends the classical Thring & Newby (1953) and Ricou & Spalding (1961) momentum diameter d^* to exothermic jet flames. In terms of d^+ the scaling laws for jets with and without heat release were seen to become identical. The equivalence principle thus accurately predicted the reduced entrainment due to heat release in turbulent jet flames, as well as the resulting effect of heat release on jet flame lengths. When this equivalence principle was applied to planar turbulent jets it led to an extended momentum width h^+ that was seen to similarly give correct predictions for the much stronger effect of heat release on the scaling laws in that flow. The equivalence also correctly predicted effects of heat release on the near-field lengths of both planar and axisymmetric turbulent jets. In particular, it indicated a much larger increase in near-field length due to heat release in planar turbulent jets than in axisymmetric jets,

in good agreement with observations and measurements in the transition region of both planar and axisymmetric turbulent jet flames.

3.2. Application to mixing layers

In mixing layers, we will let X denote the mole fraction of low-speed fluid in any mixture, so that $X_1 \equiv 0$ and $X_2 \equiv 1$. We consider an adiabatic exothermic reaction with an overall activation energy sufficiently large for the reactions to be confined to a narrow range of mole fractions around the stoichiometric value X_s , where the temperature is assumed to reach its peak value T_s . Under the conditions noted in Part 1, the temperature field $T(\mathbf{x}, t)$ will then necessarily be bilinear in terms of the mole fraction field $X(\mathbf{x}, t)$, as indicated in figure 2. We can thus equivalently represent $T[X(\mathbf{x}, t) < X_s]$ as resulting from simple mixing, without reaction, of high-speed fluid at its actual temperature T_1 with stoichiometric products at $X = X_s$ and temperature T_s . Similarly $T[X(\mathbf{x}, t) > X_s]$ is equivalent to simple mixing of stoichiometric products at temperature T_s with low-speed fluid at its actual temperature T_2 .

Following Part 1 and as indicated in figure 2, wherever $X(\mathbf{x}, t) < X_s$ the temperature $T(\mathbf{x}, t)$ in the reacting flow will be equivalent to that which would result from simple mixing without reaction between an inert low-speed stream at its actual temperature T_2 and density ρ_2 , and an inert high-speed stream at an effective elevated temperature given by

$$(T_1)_{eff} \equiv T_2 + (T_s - T_2) \frac{X_2 - X_1}{X_2 - X_s}, \quad (5a)$$

and hence at a corresponding effective density

$$(\rho_1)_{eff} = \rho_1 \left(\frac{T_1}{(T_1)_{eff}} \right). \quad (5b)$$

Both the reacting flow and the equivalent non-reacting flow will then have the same temperature fields $T(\mathbf{x}, t)$, and thus the same density fields $\rho(\mathbf{x}, t)$, wherever $X(\mathbf{x}, t) < X_s$. Similarly, wherever $X(\mathbf{x}, t) > X_s$ the temperature $T(\mathbf{x}, t)$ will be equivalent to that produced by mixing without reaction between an inert high-speed stream at its actual temperature T_1 and density ρ_1 , and an inert low-speed stream at an effective elevated temperature given by

$$(T_2)_{eff} \equiv T_1 + (T_s - T_1) \frac{X_2 - X_1}{X_s - X_1}, \quad (6a)$$

and thus at corresponding effective density

$$(\rho_2)_{eff} = \rho_2 \left(\frac{T_2}{(T_2)_{eff}} \right). \quad (6b)$$

In that case the reacting flow and the equivalent non-reacting flow will have the same temperature fields $T(\mathbf{x}, t)$, and thus the same density fields $\rho(\mathbf{x}, t)$, wherever $X(\mathbf{x}, t) > X_s$.

Under these conditions, the density field $\rho(\mathbf{x}, t)$ in the reacting flow at values of $X(\mathbf{x}, t)$ above [or below] X_s will be the same as that in the equivalent non-reacting flow with the free-stream density replaced by its effective value in (5b) [or (6b)] and with the other free-stream density kept at its actual value. Thus the effects of density changes due to heat release on the outer variables in an exothermic mixing layer should be deducible from the scaling laws that apply in the corresponding non-reacting mixing layer by simply replacing the appropriate free-stream density with its effective value.

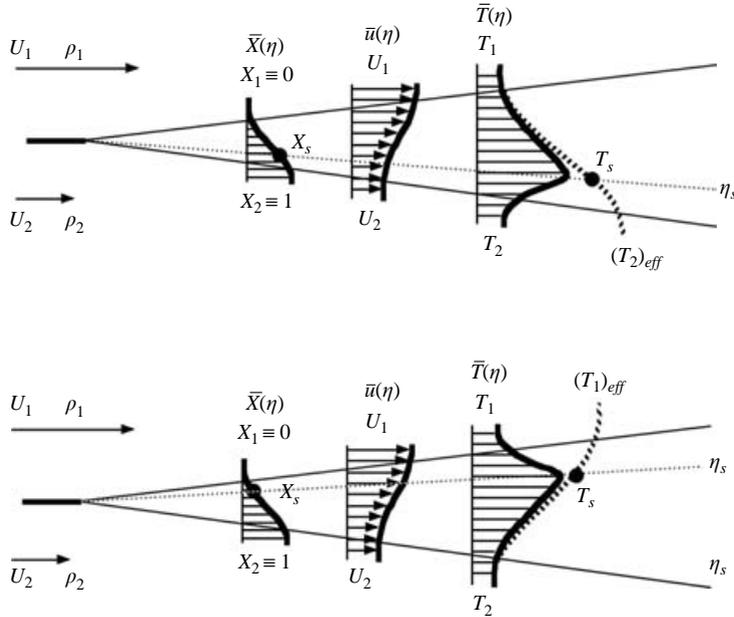


FIGURE 3. Notation for exothermic reacting turbulent mixing layers, showing mean velocity profile, mean mole fraction profile, and mean temperature profile in terms of the similarity coordinate η for the two cases discussed in §4.1. Shown is stoichiometric ray η_s where mean mole fraction is at stoichiometric value X and where mean temperature reaches its maximum value. Dashed lines show temperature profiles for equivalent non-reacting mixing layers with one free-stream temperature raised to its effective value in (5a) or (6a).

4. Scaling laws for heat release effects in mixing layers

From the considerations in §2 and §3, the scaling laws for exothermic reacting turbulent mixing layers should be obtained from the corresponding scaling laws for isothermal mixing layers under otherwise identical conditions by replacing one of the free-stream densities by its effective value in (5b) or (6b).

4.1. Effective free-stream temperature

The choice of which free stream should be considered at the equivalent elevated temperature to account for the density changes due to heat release depends on which of the two linear branches of $T(X)$ dominates the flow. For planar and axisymmetric turbulent jets considered in Part 1, the entrained flow always came from the surrounding ambient fluid. It was therefore readily apparent whether the jet fluid or the ambient fluid was to be considered at the equivalent elevated temperature, with the choice depending on whether the flow properties being considered were upstream ($\langle X \rangle > X_s$) or downstream ($\langle X \rangle < X_s$) of the mean flame tip location ($\langle X \rangle = X_s$). In the mixing layer, however, fluid is continually entrained from both free streams, and thus determination of which free-stream fluid should be considered at the equivalent elevated temperature requires more careful consideration.

Since the scaling properties of shear flows result principally from momentum conservation, the appropriate choice of which free-stream temperature to change in order to account for density changes due to heat release is that which most accurately reflects the momentum flux density profile within the layer. The two choices are shown schematically in figure 3. When $X_s \geq 1/2$, the stoichiometric ray $\eta_s \equiv y_s/(x - x_0)$, near

which the mean temperature reaches its peak value, is in the lower half of the layer. The momentum flux density is then dominated by $\eta > \eta_s$, namely the branch of $T(X)$ where $X < X_s$, and thus the momentum flux density profile is best represented by setting the temperature of the low-speed stream to $(T_2)_{eff}$ and keeping the high-speed stream at its actual temperature T_1 . Owing to the form of the mean velocity profile, this remains the case even when $X_s = 1/2$. For $X_s < 1/2$ this continues to be the case until X_s becomes sufficiently small that most of the momentum flux in the layer is accounted for by $\eta < \eta_s$. It is apparent from figure 3 that the value of X_s at which this crossover occurs, denoted X_s^* , will depend on both the speed ratio r and the stoichiometric temperature T_s . Once this crossover occurs, the momentum flux density profile is then best represented by setting the temperature of the high-speed stream to $(T_1)_{eff}$ and keeping the low-speed stream at its actual temperature T_2 . Cases in which $X_s \approx X_s^*$ would thus be expected to lead to the largest errors; however as long as X_s differs sufficiently from X_s^* the proposed equivalence should provide an accurate representation of the density field in reacting mixing layers.

4.2. Extended density ratio s^+

Thus for $X_s > X_s^*$, the two free-stream densities in the corresponding non-reacting mixing layer are ρ_1 and $(\rho_2)_{eff}$, and the appropriate extended density ratio then becomes

$$s^+ \equiv \left[\frac{(\rho_2)_{eff}}{\rho_1} \right] \quad (7a)$$

which from (6b) can be written as

$$s^+ = s \left[\frac{T_1}{(T_2)_{eff}} \right]. \quad (7b)$$

When $X_s < X_s^*$, the corresponding free-stream densities are $(\rho_1)_{eff}$ and ρ_2 , and the appropriate extended density ratio is then

$$s^+ \equiv \left[\frac{\rho_2}{(\rho_1)_{eff}} \right] \quad (8a)$$

which from (5b) can be written as

$$s^+ = s \left[\frac{(T_1)_{eff}}{T_2} \right]. \quad (8b)$$

The parameter s^+ is the 'extended density ratio' that accounts for the dominant heat release effects in exothermically reacting mixing layers. Note that for non-reacting mixing layers $s^+ = s$, while for exothermic reacting mixing layers the two differ by a factor that depends on the appropriate ratio of actual and effective free-stream temperatures from (5a) or (6a).

4.3. Scaling laws

Owing to the equivalence in § 3 and in figures 2 and 3 between the exothermic reacting flow and the corresponding equivalent isothermal flow, the scaling laws for exothermic reacting mixing layers should be obtained from those in § 2 for non-reacting mixing layers by replacing the nominal density ratio s with the appropriate extended density ratio s^+ from (7a, b) or (8a, b). Thus from (3) the growth rate for exothermic reacting

mixing layers should be

$$\frac{\delta}{(x - x_0)} \sim \left(\frac{1 - r}{1 + s^{+1/2}r} - 1 \right) \left[(1 + s^{+1/2}) - \left(\frac{1 - s^{+1/2}}{1 + 2.9(1 + r)/(1 - r)} \right) \right] \quad (9)$$

and from (4) the entrainment ratio for exothermic reacting mixing layers should be

$$E_v \approx s^{+1/2} \left(1 + 0.68 \frac{1 - r}{1 + r} \right). \quad (10)$$

In the following section, these scaling laws derived from the general equivalence principle will be evaluated by comparisons with experimentally measured heat release effects in turbulent mixing layers.

5. Comparisons with measured heat release effects

5.1. Experimental data

Wallace (1981) and Hermanson (1985) have reported experimental results for effects of heat release in incompressible exothermic reacting turbulent mixing layers. The latter results are also reported in Hermanson & Dimotakis (1989) and Hermanson *et al.* (1987), and are consistent with the earlier results of Wallace at lower levels of exothermicity. Of particular note is that tables E.1 and E.2 of Hermanson (1985) provide detailed tabulations for all experimental conditions needed to allow evaluation of the scaling laws in §4 for heat release effects in mixing layers.

Relevant parameters for all 14 cases reported by Hermanson (1985) having zero streamwise pressure gradient are shown in table 1. These include adiabatic flame temperature rises ranging from 93 K to as high as 737 K, and stoichiometric mole fractions X_s ranging from 0.50 to 0.89. Note that the stoichiometry parameter ϕ termed by Hermanson the ‘equivalence ratio’ is related to the stoichiometric mole fraction as $X_s = 1/(1 + \phi)$ for X as defined in §3.2. The experimentally measured growth rate $\delta/(x - x_0)$ was reported for each case from the measured mean temperature rise profile $\Delta T(\eta)$. All results were at nominal speed ratio $r = 0.4$ and nominal density ratio $s = 1.0$. Slight case-to-case variations in the precise (r, s) values were adjusted by Hermanson through normalization via (3) with the corresponding growth rate without heat release at the same r and s values. As noted in §1, the experimental values for $\delta/(x - x_0)$ in table 1 show only a weak effect of heat release, even at the highest adiabatic flame temperature rises, on the growth rate of mixing layers. This is in sharp contrast to both the near and far fields of both planar and axisymmetric turbulent jets considered in Part 1, where heat release effects were seen to be large. Moreover, the experimentally measured growth rates in table 1 show a *reduction* in the layer growth with increasing heat release, a result that is the opposite of what might be expected from simple displacement considerations due to the dilatation induced by heat release.

5.2. Growth rate comparisons

Table 1 also gives the resulting parameters relevant to the equivalence principle in §3 and §4, including the resulting stoichiometric mole fractions X_s and temperatures T_s , and the resulting effective free-stream temperature. Since $X_s > 1/2$ in all these cases, as discussed in §4.1 the equivalence is achieved by raising the effective temperature of the low-speed stream to $(T_2)_{eff}$. The corresponding values of the extended density ratio s^+ from (7b) are also shown in table 1, as are the resulting predicted growth rates $\delta/(x - x_0)$ from (9). These predicted growth rates have also been normalized via

Run no.	From tables E.1 and E.2 of Hermanson (1985)									From equivalence principle based on T_s			From equivalence principle based on T_M		
	U_1 (m s ⁻¹)	U_2 (m s ⁻¹)	r (—)	s (—)	ϕ (—)	T_s (K)	X_s (—)	$\frac{\delta}{(x - x_0)}$	\bar{T}_M (K)	$(T_2)_{eff}$ (K)	s^+ (—)	$\frac{\delta}{(x - x_0)}$	$(T_2)_{eff}$ (K)	s^+ (—)	$\frac{\delta}{(x - x_0)}$
34	22.66	8.78	0.387	1.013	1.00	393.0	0.50	0.158	327.9	486.0	0.625	0.155	355.8	0.854	0.162
35	22.67	8.88	0.392	1.023	0.50	424.0	0.67	0.155	337.2	486.0	0.631	0.155	355.8	0.863	0.162
37	23.25	8.79	0.378	1.084	0.13	465.0	0.88	0.155	349.5	485.6	0.670	0.155	355.7	0.914	0.162
38	22.89	8.98	0.392	1.042	0.25	449.0	0.80	0.154	344.7	486.3	0.643	0.155	355.9	0.878	0.162
88	24.04	8.80	0.366	1.183	0.13	628.4	0.88	0.155	398.5	669.5	0.530	0.148	410.8	0.864	0.158
90	22.01	8.21	0.373	1.046	0.50	547.6	0.67	0.152	374.3	671.4	0.467	0.148	411.4	0.763	0.158
91	20.91	8.20	0.392	1.040	1.00	578.1	0.50	0.149	383.4	856.2	0.364	0.145	466.9	0.668	0.156
119	21.15	8.32	0.393	1.067	1.00	756.8	0.50	0.148	437.0	1213.6	0.264	0.139	574.1	0.558	0.152
123	21.17	7.90	0.373	1.000	0.25	900.3	0.80	0.150	480.1	1050.4	0.286	0.140	525.1	0.571	0.153
125	21.04	8.00	0.380	1.026	1.00	486.3	0.50	0.147	355.9	672.6	0.458	0.149	411.8	0.747	0.159
130	21.23	8.12	0.382	1.000	1.00	853.8	0.50	0.147	466.1	1407.6	0.213	0.136	632.3	0.474	0.150
150	21.54	8.15	0.378	1.000	0.50	796.3	0.67	0.149	448.9	1044.5	0.287	0.141	523.3	0.573	0.154
152	21.65	8.30	0.383	1.000	0.25	601.9	0.80	0.150	390.6	677.4	0.443	0.149	413.2	0.726	0.159
157	21.94	7.99	0.364	1.000	0.50	1037.4	0.67	0.149	521.2	1406.1	0.213	0.135	631.8	0.475	0.149

TABLE 1. Conditions from Hermanson's (1985) measurements of heat release effects in incompressible turbulent reacting mixing layers, showing measured growth rates $\delta/(x - x_0)$ together with results from (6a), (7b) and (9) from the equivalence principle based on matching to T_s and T_M in figure 5.

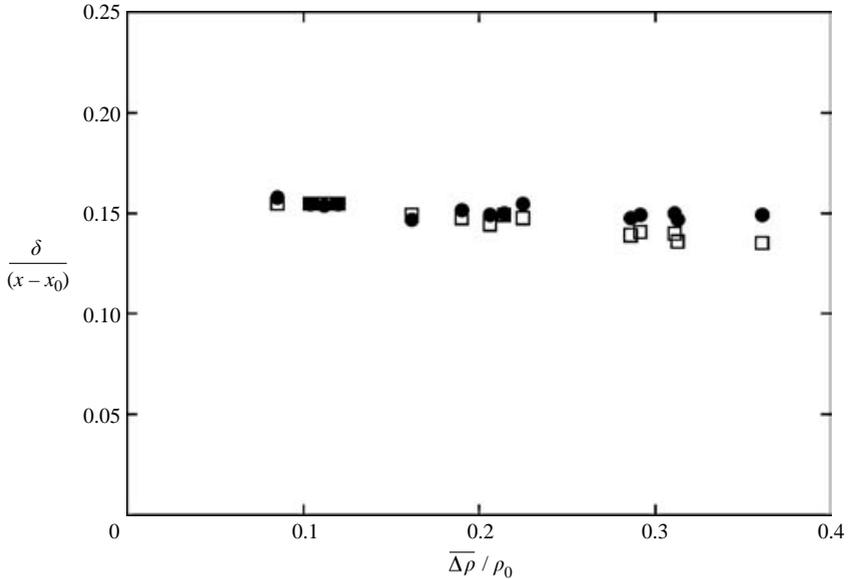


FIGURE 4. Comparison of measured growth rate (solid symbols) from Hermanson (1985) and Hermanson & Dimotakis (1989) with predicted growth rate (open symbols) from (6a, b), (7a, b) and (9) based on the equivalence principle indicated in figure 3.

(3) to $r = 0.4$ and $s = 1.0$ to remove the slight case-to-case variations in r and s , and thereby allow direct comparisons with the similarly normalized results of Hermanson (1985).

The resulting comparisons are shown in figure 4. Note that the predicted growth rates from (6a, b), (7a, b) and (9) also show only a weak effect of heat release on the mixing layer, consistent with the observations of Hermanson (1985) and Hermanson & Dimotakis (1989), as well as the earlier observations of Wallace (1981) at lower levels of heat release. Moreover, the predicted growth rates from this equivalence principle also indicate that the effect of increasing heat release rate in turbulent mixing layers is to *reduce* the growth rate of the layer, in direct opposition to what simple considerations based on the displacement effect might suggest.

While both the trend and the magnitude of the heat release effect in mixing layers are correctly predicted by the simple equivalence principle in § 3, figure 4 indicates that the equivalence as implemented above slightly overpredicts the effect of heat release. The reason for this is apparent in figure 2, where the extension of the linear part of $T(X)$ for $X < X_s$ to higher values of X necessarily leads to excessively high values of the mean temperature rise within the flow. The result is a slight overprediction of the heat release effect, even though the overall trend and magnitude of the effect and its parametric dependences are correctly accounted for. This may be readily shown as indicated in figure 5 by using the simple mixing line in $T(X)$ that matches the peak mean temperature T_M at $X = X_s$, rather than matching to T_s . This simply replaces T_s in (5)–(8) with the measured T_M values reported by Hermanson (1985) and shown in table 1. These produce the growth rate comparisons shown in figure 6, where it is apparent that the simple equivalence principle correctly captures the effects of heat release in the mixing layer.

These results strongly suggest that the physical mechanisms which produce the heat release effect on the growth rate of turbulent mixing layers are the same as those

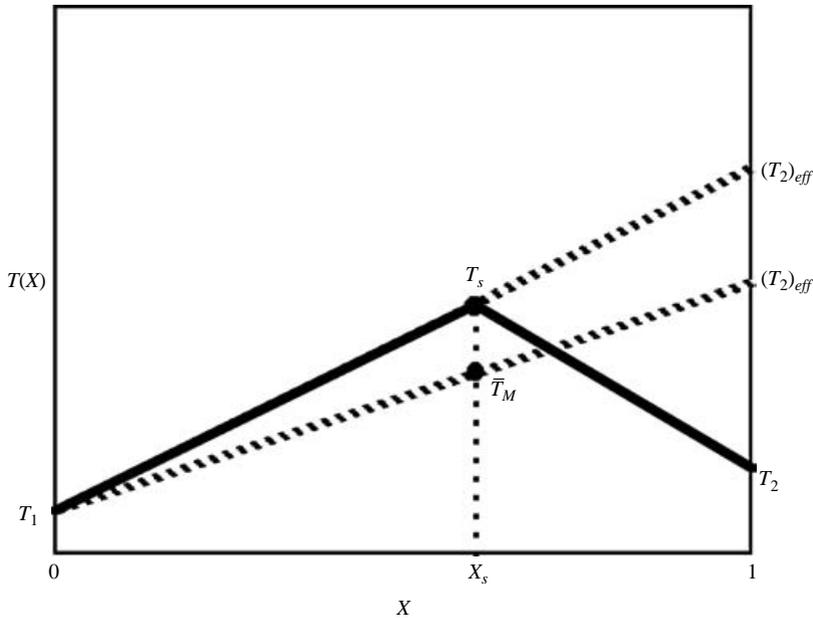


FIGURE 5. Alternative mixing line in $T(X)$ matching true mean temperature at the stoichiometric mole fraction X_s , yielding lower effective free-stream temperature $(T_2)_{eff}$ that eliminates overprediction of heat release effect.

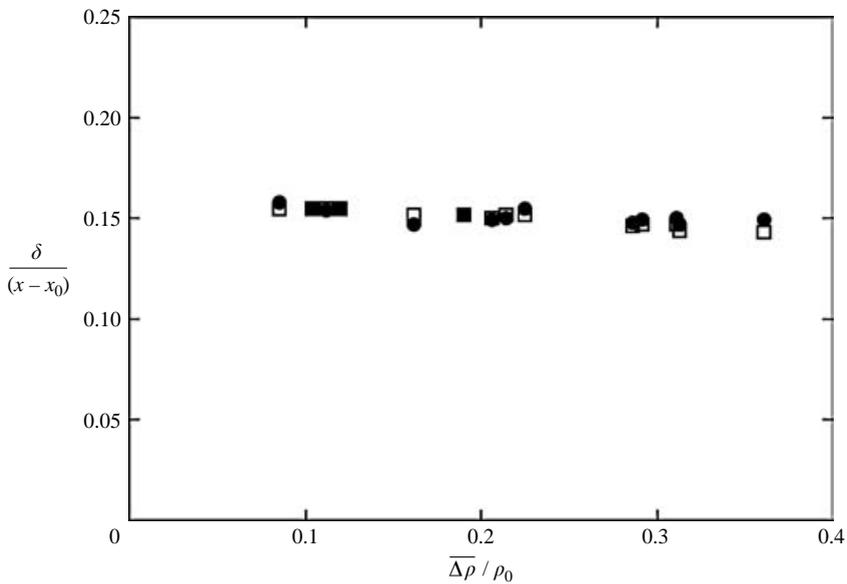


FIGURE 6. Comparison of measured growth rates (solid symbols) from Hermanson (1985) and Hermanson & Dimotakis (1989) with predicted growth rates (open symbols) from (6a, b), (7a, b) and (9) from the equivalence principle by matching to true mean temperature as indicated in figure 5. Much of the apparent scatter in the measured values is largely replicated by predicted values from the equivalence principle, indicating a weak parametric effect of stoichiometry accounted for in the equivalence principle.

which produce the effect of free-stream density differences in non-reacting mixing layers. That conclusion becomes even more forceful when the above heat release effects, obtained from application of the equivalence principle in mixing layers, are viewed together with the heat release effects obtained in the near and far fields of both planar and axisymmetric turbulent jets in Part 1. Despite the widely differing nature of the individual heat release effects and their relative magnitudes among these various flows, it is apparent that this simple equivalence principle correctly predicts the heat release effects in all of these flows based only on the proposed adjustment of the appropriate free-stream temperature to account for the temperature rise within the flow due to exothermicity.

5.3. Stoichiometry effects

Close examination of the points in figures 4 and 6 shows that in many cases the apparent scatter in the experimentally measured growth rates is replicated point-for-point by the predicted values from the equivalence principle. This is especially apparent in figure 6, and suggests that the scatter is a reflection of a weaker parametric effect on the heat release effect in mixing layers. From §3 and §4 the effect may be due either to the stoichiometric mole fraction X_s or to the temperature T_s . While Hermanson & Dimotakis (1989) state that they observed no dependence of the thinning trend on stoichiometry at a given value of $\overline{\Delta\rho}/\rho_0$, the equivalence principle suggests that there should in fact be such an effect. This can be readily understood from figure 3. For a given value of T_s , any shift in the stoichiometric ray η_s will lead to a relatively small change in the value of $(T_2)_{eff}$, which in turn will produce a change in s^+ via (7b) and thus a change in the growth rate $\delta/(x-x_0)$ via (9). The equivalence principle allows even these relatively weak parametric effects to be predicted. In particular, figure 7(a, b) shows the relative effects of X_s and T_s on the growth rate of the mixing layer when all other parameters are held constant at various values. These results confirm a relatively weak effect of the stoichiometry on the growth rate of the mixing layer, consistent with the apparent 'scatter' in the results of Hermanson & Dimotakis (1989).

5.4. Heat release effects on entrainment ratio

Hermanson (1985) does not tabulate measured values for the entrainment ratio E_v for each of his cases; however figure 8 of Hermanson & Dimotakis (1989) reports a reduction in the entrainment ratio with increasing heat release. In particular, they suggest a reduction in E_v of roughly 15% for a mean density reduction $\overline{\Delta\rho}/\rho_0$ of 40%, though there is considerable variation apparent in their data, with some of the points showing reductions nearly twice as large per unit density reduction. Based on the considerations in §5.3 much of this apparent scatter may be due to stoichiometry effects; however, lacking tabulated values this cannot be verified. Additionally, there are ambiguities in their calculation of E_v that further complicate any direct comparison with the values in their figure 8. Nevertheless, based on (7b) and (10) the equivalence principle suggests that the entrainment ratio E_v will change with increasing heat release as

$$\frac{E_v}{(E_v)_0} = \left(\frac{s^+}{s}\right)^{1/2} = \left(\frac{T_2}{(T_2)_{eff}}\right)^{1/2}. \quad (11)$$

From the values of $(T_2)_{eff}$ for each case in table 1 based on the linear $T(X)$ that matches the peak mean temperature rise in the mixing layer in figure 5, the resulting predicted values of the entrainment ratio E_v from (11) are shown in the present

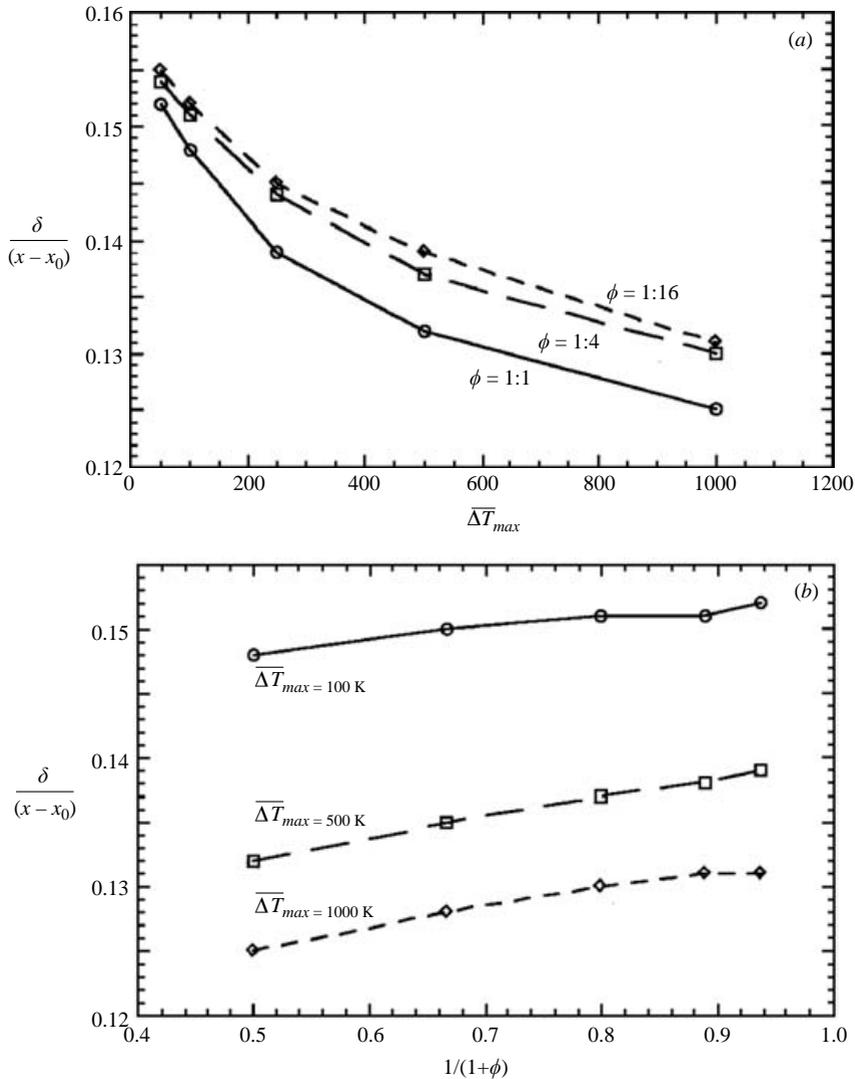


FIGURE 7. Effects of (a) flame temperature for fixed stoichiometry and (b) stoichiometry for fixed flame temperature, showing relatively weak parametric effects of heat release in mixing layers implied by the equivalence principle.

figure 8. These results are in qualitative agreement with those in figure 8 of Hermanson & Dimotakis (1989), showing a significant decrease in entrainment ratio with increasing heat release. Given the uncertainties in the results of Hermanson & Dimotakis, including measurement of the virtual origin x_0 , a more precise comparison may not be possible, but the equivalence principle based simply on accounting for the density reductions in the mixing layer correctly predicts a reduction in the entrainment ratio due to heat release.

6. Discussion and conclusions

This study has applied the equivalence principle of Tacina & Dahm (2000) to develop scaling laws for heat release effects produced by nonpremixed or partially

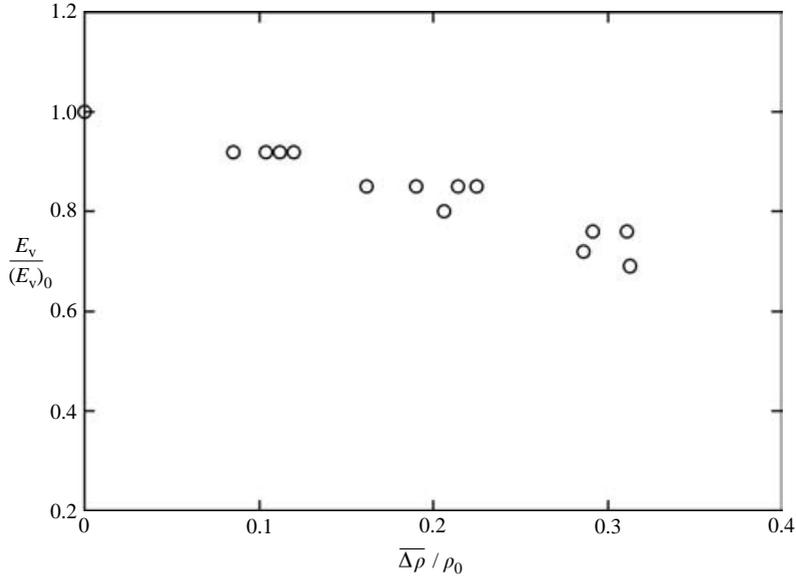


FIGURE 8. Reduction in entrainment ratio E_v with heat release from (6a, b), (7a, b) and (10) based on the equivalence principle, showing qualitatively similar reduction due to heat release as measured by Hermanson (1985) and Hermanson & Dimotakis (1989).

premixed combustion in incompressible turbulent mixing layers in the absence of a streamwise pressure gradient. Density effects play a direct role in the scaling of the growth rate and entrainment properties of mixing layers, even in the absence of heat release, and thus when heat release is present the resulting density changes may be expected to affect the flow through these scaling laws. In particular, the equivalence principle allows the density changes due to exothermic reaction to be related to an equivalent non-reacting mixing layer, in which the temperature of one of the free-stream fluids is raised to an effective value determined by the peak temperature and overall stoichiometry. This leads to a generalized density ratio s^+ that provides scaling laws for heat release effects in exothermic reacting mixing layers. The resulting predicted effects of heat release in mixing layers show good agreement with experimentally measured values, and reveal additional subtle effects of stoichiometry previously unnoticed in the experimental results. While the equivalence is restricted to essentially adiabatic flows and activation energies sufficiently large for the reactions to be confined to a narrow range of mole fractions around the stoichiometric value, these conditions are applicable in many practical combustion applications.

The scaling laws obtained here for heat release effects in turbulent mixing layers support the validity and utility of the general equivalence principle of Tacina & Dahm (2000). These scaling laws allow prediction of heat release effects in practical situations, and in addition allow insights into parametric influences on heat release effects in mixing layers. Moreover, these results permit new insights into possible ways to derive potentially desirable benefits from the heat release effect in mixing layers.

For example, it is apparent from §4 that the reduction in s^+ relative to s leads to the decreased growth rate, entrainment rate, and entrainment ratio seen in §5 and in the experiments of Wallace (1981), Hermanson & Dimotakis (1989) and Mungal & Dimotakis (1985). Note, however, that if the velocity ratio, stoichiometry, and flame temperature are arranged so that X_s switches from $X_s > X_s^*$ to $X_s < X_s^*$, as described

in §4.1, then the effective free-stream temperature switches from the low-speed to the high-speed stream, and consequently s^+ switches from (7a, b) to (8a, b). The result is that the effect of heat release switches from $s^+ < s$ to $s^+ > s$, with the consequences from (9) and (10) being to switch from a *reduction* in the growth rate and entrainment ratio due to heat release to an *increase* in these properties of the mixing layer due to heat release. This somewhat surprising result, which stands in stark contrast to the widely held belief that heat release in mixing layers always acts to reduce the growth rate and entrainment ratio, follows directly from the present application of the equivalence principle to mixing layers. Indeed, while this potential means for exploiting heat release effects to produce increased growth rates and entrainment ratios must currently be viewed as speculative, there is some evidence in the results of Mungal & Dimotakis (1985) at moderate heat release to suggest that this prediction may be correct. In particular, their table 2 includes cases with adiabatic flame temperature rise of 165 K and stoichiometric ratios of $\phi = 1/8$ and 8, corresponding respectively to $X_s = 8/9$ and $1/9$. It is not apparent if the latter X_s value is sufficiently small for the switching to occur, since the relatively large velocity difference ($r = 0.4$) suggests that more momentum flux may still be carried by the flow above the stoichiometric ray in figure 3. However the corresponding growth rates in their table 1 do indeed show a greater layer thickness for the lower X_s value. In fact, their results consistently show a much larger change in the growth rate when $\phi > 1$ than when $\phi < 1$, consistent with the higher values of $(T_2)_{eff}$ indicated in figure 3 by the equivalence principle.

The present results for heat release effects in mixing layers provide further strong support for the general equivalence principle of Tacina & Dahm (2000) between exothermic reacting turbulent shear flows and a corresponding equivalent non-reacting flow under otherwise identical conditions. These results strongly suggest that the physical mechanisms which produce the heat release effect on the growth rate of turbulent mixing layers are the same as those which produce the effect of free-stream density differences in non-reacting mixing layers. That conclusion becomes even more forceful when the above heat release effects in turbulent mixing layers are viewed together with the results obtained in Part 1 for effects of heat release in both the near and far fields of both planar and axisymmetric turbulent jets flames over a wide range of fuels and dilutions. Despite the widely differing nature of the individual heat release effects and their relative magnitudes in all of these flows, it is apparent that this simple equivalence principles correctly predicts these effects based only on a physically based adjustment of one of the free-stream temperatures to account for the temperature rise within the flow due to exothermicity, and thereby readily provides scaling laws for heat release effects in shear flows.

As noted in Part 1, the present equivalence fails if used outside the range over which its physical assumptions apply. This includes cases in which buoyancy due to heat release is no longer negligible, as well as cases in which the increase in viscosity due to heat release produces a sufficient reduction in Reynolds number for the flow to become transitional or laminar. Similarly, in strongly radiating cases or in cases with large heat extraction, the underlying adiabatic assumption will no longer allow the temperature to be adequately determined by the mole fraction. A similar limitation applies if the dominant heat-releasing reactions are not sufficiently fast to avoid significant chemical non-equilibrium effects, as may occur in very high-speed flows with significant local extinction. Moreover, if temperatures in the reacting flow are high enough for dissociation effects to become significant, as is often the case in oxygen-enriched combustion, then $T(X)$ will no longer be piecewise linear. However, in most practical combustion applications, the equivalence principle will be applicable

and the scaling laws derived here from it will accurately determine the changes due to heat release in the entrainment and mixing properties of nonpremixed or partially premixed exothermic reacting turbulent mixing layers.

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